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No. 1.

Rev. CHARLES PRITCHARD, President, in the Chair.

Wm. Westgarth, Jun., Esq., 5 Hadley Street, Kentish Town;

Rev. J. Maurice Wilson, Rugby School;

Rev. Frank Besant, Stockwell Grammar School;

E. B. Denison, Esq. LL.D. Q.C., 33 Queen Anne Street;

Rev. N. Jennings, Avenue Road, Regent's Park;

George Manners, Esq., Lansdown Road, Croydon; and

Capt. R. M. Parsons, R.E., Ordnance Survey Office, Southampton,

were balloted for and duly elected Fellows of the Society.

On the Mass of Jupiter, as deduced by Herr Krüger from the Observations of Themis. By W. T. Lynn, Esq.

The Astronomer Royal having kindly placed in my hands a paper containing an investigation recently made by Herr Krüger of the orbit of *Themis*, with special reference to re-determining the mass of *Jupiter* by the effects its attraction produces on that planet, with a request that I would communicate something of its contents to the Royal Astronomical Society, I very gladly avail myself of this opportunity of doing so. And it appears desirable that I should commence with a brief review of the state of our knowledge with regard to that important element in the solar system, the mass of the planet *Jupiter*, its largest member with the exception of the central luminary itself. What that knowledge was in the year 1832, cannot be more clearly stated than in the words of Mr. Airy,

Mr. Lynn, on the Mass of Jupiter,

then Plumian Professor at Cambridge, to the Meeting of the British Association held that year at Oxford. He said, "In the *Berlin Ephemeris* for 1826, Nicolai gave a short paper containing results of great importance deduced from the discussion of observations on *Juno*. In all the calculations hitherto made, the mass adopted for *Jupiter* was either that assumed by Laplace (founded on Pound's observations of the elongations of *Jupiter's* satellites), or that given by Bouvard (from the perturbations of *Saturn*), differing little from the other. Now Nicolai stated that the observations of *Juno* at fifteen oppositions required an increase of about $\frac{1}{80}$ th in the mass of *Jupiter*; but that even then the observations could not be well represented; and that he conceived the *absolute attraction* of *Jupiter* on *Juno* must be different from that upon the Sun. The last conclusion, attacking one of the most important principles in the theory of gravitation, required further examination. In the *Berlin Memoirs*, 1826, Encke discussed all the observed oppositions (fourteen) of *Vesta*, separating the perturbations produced by *Jupiter* into two parts, one being *Jupiter's* attraction on the Sun and the other *Jupiter's* attraction on *Vesta*, and considering the assumed mass of *Jupiter* in these two attractions as liable to two separate errors. The result was, that the absolute attraction of *Jupiter* on *Vesta* did not differ from that on the Sun by more than $\frac{1}{10000}$ th of the whole, and that Nicolai's mass ought to be increased about $\frac{1}{300}$ th of the whole. Encke remarks, however, that Nicolai's mass will represent the observations very nearly as well; and Gauss has found the same for *Pallas*. Nicolai's mass is generally adopted by the German astronomers." The mass obtained by Nicolai was $\frac{1}{1053'921}$; that by Encke $\frac{1}{1050'36}$.

Since the time spoken of in the Report from which I have quoted,* two excellent determinations of *Jupiter's* mass from the motions of his satellites have been made, the first by Airy, the second by Bessel. The observations used by the former were made partly at Cambridge, and partly at Greenwich. The author remarks in the first of his three papers on the subject (*Memoirs of the R. A. S.*, vol. vi., p. 83), that, considering the great importance of the mass of *Jupiter* in both planetary and cometary investigations, "it seems remarkable that no attempts should lately have been made to ascertain the correct value from the readiest of all means, the elongations of his satellites. It is true that the value adopted by Laplace, $\frac{1}{1067'09}$ of the Sun's mass, was inferred from observations of the

* It may be mentioned that at the close of that Report, Mr. Airy, amongst other suggestions of points to which attention should be directed, recommends, what he afterwards himself carried out, that new measurements be made of the elongations of *Jupiter's* satellites, especially of those of the fourth.

fourth satellite; but they were observations made by Pound, and of which no account (so far as I know) remains, except the mere statement of numbers in the *Principia*." Mr. Airy concludes, at the end of this paper, that "Nicolai's mass cannot now be admitted, but Encke's mass is nearly as well supported as mine by these observations."

The mass he then determined was $\frac{1}{1048.70}$, a little larger than Encke's; and this, from a discussion of a larger number of observations, he afterwards (*Memoirs*, vol. x. p. 47) still further increased, his final value being $\frac{1}{1046.77}$. This result was communicated to the Society in January 1837. The confirmation afforded to this by the determination of Bessel from his Königsberg observations is very striking. His value is $\frac{1}{1047.87}$, and he remarks in regard to it, in a paper communicated to the French Academy of Sciences (*Comptes Rendus*, 1841), "Je suis enfin parvenu à la connaissance d'un élément d'observation bien important, et j'ose me flatter qu'une erreur plus grande que $\frac{1}{10,000}$ est impossible." He also mentions its close agreement with Airy's value.

In the paper to which I have now the honour of calling the attention of the Society, and which has recently been communicated to the Finnish Society of Sciences, Krüger states that he has been for some time engaged on the theory of *Themis*, "principally in view of the valuable material which that planet must in time afford for the determination of the attraction of *Jupiter*." The opinion has in recent times been expressed by Schubert that Bessel's mass, which, as we have seen, agrees so closely with Airy's, and which is now adopted in nearly all calculations, is in reality further from the truth than the smaller value deduced by Nicolai from the oppositions of *Juno*; his ground for this being, in addition to what he had noticed in his own calculations, the fact that Brünnow, in discussing the motions of *Iris*, had also come to the same result,—a diminution of the mass of *Jupiter* that had been employed.* The point becomes, as Krüger justly remarks, very interesting, since it would seem to involve the question first raised by Nicolai, whether, if the mass of *Jupiter* deduced from the motions of the small planets and from those of his own satellites, differed by any decided amount, it would be necessary to assume the existence of some forces hitherto unknown or imperfections in the application of the theories on which those motions are calculated. It is satisfactory, then, to find that the result of his elaborate investigation is, that, instead of diminishing Bessel's value of the mass of *Jupiter*; it ought really to

* Schubert's remarks on the subject are contained in *Ast. Nach.* No. 1562.

be increased by the $\frac{68}{100,000}$ th part of its value,—a quantity so small that we may regard it as tending to the complete confirmation of that mass. I will endeavour to give the Society a sketch of the way in which Krüger has treated the observations.

The method pursued is Encke's method of mechanical quadratures; and he begins by remarking that, with whatever accuracy the calculation may be made, the decimal figures which are not included must, or at least may, induce small errors in the perturbations of the co-ordinates and their differential quotients, which errors will go on increasing with the time: nor is this affected by the transition to new osculating elements. Thus the method will cease to be sufficiently accurate if long intervals of time are taken into account. But the errors induced are essentially diminished if the investigation be conducted by means of well-selected polar co-ordinates. "I have, as yet, however," says the author, "deviated in no respect from Encke's method, partly from unwillingness to interrupt calculations once begun, and partly that I might not lose the advantage of the convenient and time-saving co-ordinate tables now published. But should a repetition of the calculation of the perturbations hereafter appear desirable, I think that I should then feel bound to give the preference to the method of polar co-ordinates."

He adds that the investigation of the absolute perturbations would, if performed with the degree of accuracy necessary to draw conclusions concerning the mass of the disturbing planet, be an excessively laborious and tedious operation.

He sets out from a set of elements for *Themis* in 1853, and uses perturbing forces for *Jupiter* and *Saturn* (the effects due to each planet being simultaneously calculated and combined throughout) derived from Bessel's values of the masses of those planets. In the mechanical quadrature, the calculation was extended to the ninth decimal. A new set of elements was deduced, which, even for the last oppositions in 1864 and 1865, furnish places differing by not more than about one second of time from those observed. Continuing the calculation of the perturbations, the author made use of the co-ordinate tables which, since 1858, have been communicated in the *Astronomische Nachrichten*, and obtained new osculating elements for 1864. Krüger then compares places derived from the sets of elements he has obtained with observed places taken from the *Astronomische Nachrichten* and our *Monthly Notices*, referring the normal places to the ecliptic on account of the small inclination of the planet, and proceeds to show how he has calculated a co-efficient for the improvement of the mass of *Jupiter*. "All the perturbations," he says, "previously given, refer to the united influence of *Jupiter* and *Saturn*. But since the attraction of the latter, as is known *à priori*, can only be insignificant, and particularly

in the case before us, can only amount to a small fraction of that of *Jupiter*, it will be allowed that the correction-factor for determining the perturbations from the longitude-equations must be regarded as referring to the mass of *Jupiter* alone, because, owing to its smallness, its multiplication into the perturbations of *Saturn* can only give imperceptibly small quantities." The author feels bound to consider also the perturbations produced by *Mars*, although it is easy to show that that planet has but a trifling effect on the motions of *Themis*, yet, owing to the near commensurability of their mean motions, there arises a perturbation-term of long period, which in so short a time as twelve years, is insignificant, but in a longer interval would have to be taken into account. With the exception of some places in 1860, derived from observations made at Berlin, and which Krüger thinks are affected by some error in the elements of the very faint stars of comparison made use of, the agreement he obtains between the calculated and observed places is very close. The uncertainty in those places cannot appreciably affect his final result which I have previously mentioned, "*that the observations of Themis require an augmentation of Bessel's mass of Jupiter, amounting to the $\frac{68}{100000}$ -th part of its value.*" This gives for the actual mass of *Jupiter* compared with the Sun, the fraction $\frac{1}{1047.158}$, almost exactly equal to the mean of the values determined by Airy and Bessel from the motions of the satellites.

Observations of the New Variable T Coronæ.

By J. Baxendell, Esq.

Since Mr. Huggins communicated my early observations of this star to the Society (*Monthly Notices*, vol. xxvi. p. 277), I have observed it as often as the state of the weather and other circumstances would permit; and having lately carefully re-determined the magnitudes of the stars with which I have compared it during its changes, and made a fresh reduction of all my observations, I now submit the results to the Society, believing that a projection of the entire series will give as correct a representation of the course of the star's changes up to the present time as can well be obtained. All the observations after the star became invisible to the naked eye were, with only three exceptions, made at Mr. Worthington's Observatory with his equatorially mounted achromatic of 5 inches aperture, and always with the same eyepiece, a positive, having a magnifying power of 68 times.

In the scale of magnitudes which I have employed the